Application of wavelet technique for comparison of centre of mass time series determined by SLR, GNSS and DORIS techniques

W. Kosek[1,2], W.Popiński[2], A.Wnęk[1], M. Zbylut[1]

[1]Environmental Engineering and Land Surveying, University of Agriculture in Krakow, Poland

[2]Space Research Center, Polish Academy of Sciences, Warsaw, Poland

ABSTRACT

Geocenter motion is the motion of center of mass with respect to the center of figure of the Earth. The geocenter time series can be now computed from the Satellite Laser Ranging (SLR), Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) and Global Navigation Satellite Systems (GNSS) observations with the sampling interval ranking from one day to one week. The time-frequency wavelet spectra computed from Morlet wavelet transform coefficients of these time series reveal seasonal and subseasonal oscillations in these data. The wavelet semblance function was applied to compute time-frequency correlation coefficients between these 3D time series projected as 2D complex-valued time series onto XY, YZ, and ZX planes of the International Terrestrial Reference Frame (ITRF). Additionally, the polarization functions computed from wavelet spectra enabled detection of common variations in the flattening and polarization of the relevantelliptic oscillations in these data.

INTRODUCTION

The ITRF, used as a reference for determining e.g. precise satellite orbits, mean sea level (MSL) rise and tectonic plate motion, should be determined with the highest possible accuracy. It is defined by the origin, scale and axes orientation. Actually, the precision of the ITRF is significantly influenced by determination of its origin which is defined as the mean Earth center of mass, averaged over the whole time span of the Satellite Laser Ranging (SLR) observations used and modelled as a secular (linear) function of time [Petit and Luzum, 2010].

The center of mass of the Earth (CM) is the center of mass of the whole Earth including atmosphere, oceans and continental waters and therefore is distinguished from the center of figure (CF) of the solid Earth. The motion of the CM with respect to the CF is defined as geocenter motion (other authors use the term geocenter motion as the motion of the CF with respect to the CM, e.g. [Dong et al., 2003]). The origin of the ITRF is recognized as the CF and the geocenter motion is acknowledged as motion of the CM with respect to the origin of the ITRF [Cheng et al., 2010].

Geocenter motion time series are observed by stations located on the surface of the Earth, which track satellites orbiting around the CM. Despite the fact that the origin of the ITRF is determined only on the basis of SLR observations, the geocenter time series are computed also from observations of the DORIS and GNSS techniques. These techniques provide time series of coordinates of the origin of the particular terrestrial reference frames. This origin is situated at the center of mass of the Earth at the mean epoch of measurements and serves as CF. Consequently, such CF appointed from each technique can be used to define the origin of the ITRF. Geocenter motion is caused by mass redistribution within the Earth which entails changes in gravitational and surface forces. Therefore another method of observing this motion is computation of first degree spherical harmonic coefficients as the part of Earth's gravitational potential model [Blewitt et al., 2010; Cheng et al., 2010]. Geocenter motion can be also observed and analyzed by using geophysical and climate models of ocean tides, circulation, atmospheric and land hydrology. Nevertheless, the accuracy of these measurements is not sufficient for many modern geodetic applications [Wu et al., 2012].

THE WAVELET TRANSFORM SPECTRUM

The wavelet transform (WT) coefficient of complex-valued time series $x(t)$, $t = 0,1,...,n-1$ is defined as:

$$\hat{X}(b,a) = \sqrt{|a|}\frac{1}{n}\sum_{\omega=-n/2+1}^{n/2}\breve{x}(\omega)\overline{\breve{\psi}}(a2\pi\omega/n)\exp(i2\pi b\omega/n),$$

where $a \neq 0$ and $b = 0,1,...,n-1$ - the dilation and the translation parameters, respectively,

$\breve{x}(\omega)$, $\omega = -n/2+1,...,n/2-1,n/2$ - the Discrete Fourier Transforms (DFT) of $x(t)$ time series

$\breve{\psi}(\omega)$ - the Continuous Fourier Transform (CFT) of the modified Morlet wavelet function given by the time domain formula (Schmitz-Hübsch and Schuh 1999):

$$\psi(t) = \frac{1}{\sqrt{2\pi}}\exp(i2\pi t)\left[\exp(-t^2/2\lambda^2) - \sqrt{2}\exp(-t^2/\lambda^2)\exp(-4\pi^2\lambda^2/4)\right].$$

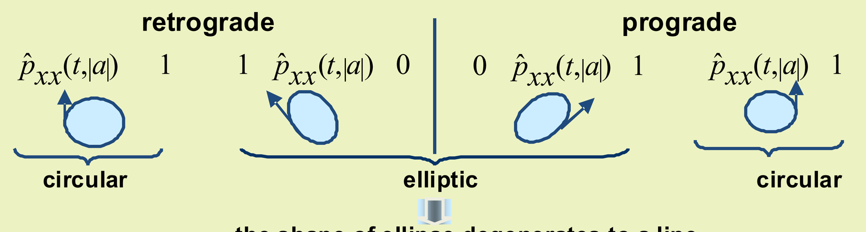
The WT spectrum is defined as:

$$\hat{S}_{xx}(t,a) = \sum_{b=-m/2+1}^{m/2}\left|\hat{X}(t-b,a)\right|^2/m, \quad t = m/2-1,...,n-1-m/2, \quad 2 \le m \le n-1.$$

THE WAVELET TRANSFORM POLARIZATION

The WT polarization is computed from the WT spectrum by the following formula:

$$\hat{p}_{xx}(t,|a|) = \frac{\hat{S}_{xx}(t,|a|) - \hat{S}_{xx}(t,-|a|)}{\hat{S}_{xx}(t,|a|) + \hat{S}_{xx}(t,-|a|)},$$



THE WAVELET TRANSFORM SEMBLANCE

The WT semblance of the order $r = 1,3,5...$, between $x(t)$ and $y(t)$, $t = 0,1,...,n-1$, time series is defined for $2 \le m \le n-1$ as:

$$\hat{\sigma}^r_{xy}(t,a) = \hat{\gamma}_{xy}(t,a)\cos^r(\hat{\theta}_{xy}(t,a)), \quad t = m/2-1,...,n-1-m/2, \quad 2 \le m \le n-1,$$

where $\hat{\gamma}_{xy}(t,a) = \left|\hat{S}_{xy}(t,a)\right| / \sqrt{\hat{S}_{xx}(t,a)\hat{S}_{yy}(t,a)}$ - the WT coherence,

$\hat{\theta}_{xy}(t,a) = \arg\frac{1}{m}\sum_{b=-m/2+1}^{m/2}[\hat{X}(t-b,a)\overline{\hat{Y}(t-b,a)}]/\left|\hat{X}(t-b,a)\right|\left|\hat{Y}(t-b,a)\right|$ - the WT phase synchronization,

$\hat{S}_{yy}(t,a) = \sum_{b=-m/2+1}^{m/2}\left|\hat{Y}(t-b,a)\right|^2/m$, - the WT spectrum of $y(t)$ time series,

$\hat{S}_{xy}(t,a) = \sum_{b=-m/2+1}^{m/2}\hat{X}(t-b,a)\overline{\hat{Y}(t-b,a)}/m$ - the WT cross-spectrum between $x(t)$ and $y(t)$ time series.

$\hat{Y}(b,a) = \sqrt{|a|}\frac{1}{n}\sum_{\omega=-n/2+1}^{n/2}\breve{y}(\omega)\overline{\breve{\psi}}(a2\pi\omega/n)\exp(i2\pi b\omega/n)$ - the WT coefficient of $y(t)$,

$\breve{y}(\omega)$, $\omega = -n/2+1,...,n/2-1,n/2$, - the DFT of $y(t)$ time series.

DATA

- SLR: GEOC94-12.GCC; Sos'nica, K., D. Thaller, A. Jäggi, R. Dach, G. Beutler; 2011: *Reprocessing 17 years of observations to LAGEOS-1 and -2 satellites.* Geodätische Woche 2011, Nürnberg, Germany, September 26-29, 2011, http://www.bernese.unibe.ch/publist/2011/pres/ks_Geod_Woche.pdf
- DORIS: ign09wd01.geoc; DORIS IGN/JPL geocenter time series available at CDDIS from 1993.01 to 2011.43, ftp://cddis.gsfc.nasa.gov/pub/doris/products/geoc/
- GNSS: 5-4_igs.sum; aparent geocenter IGS weekly combined solution from 1994.0 to 2011.90 ,ftp://igs-rf.ign.fr/pub/sum/

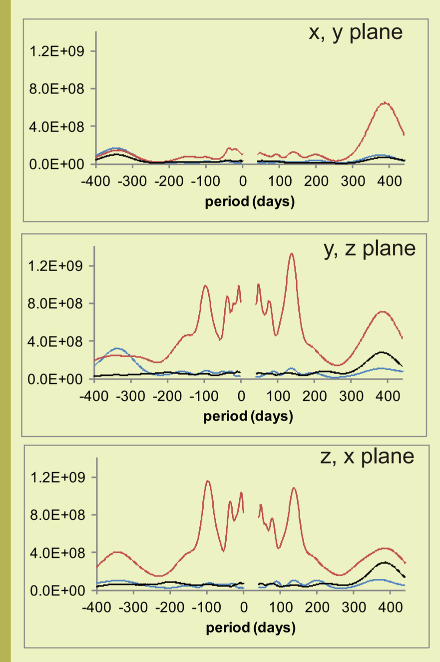


Fig. 1. The wavelet spectra (1.0, m=n) of the DORIS, GNSS and SLR center of mass time series in xy, yz and zx planes.

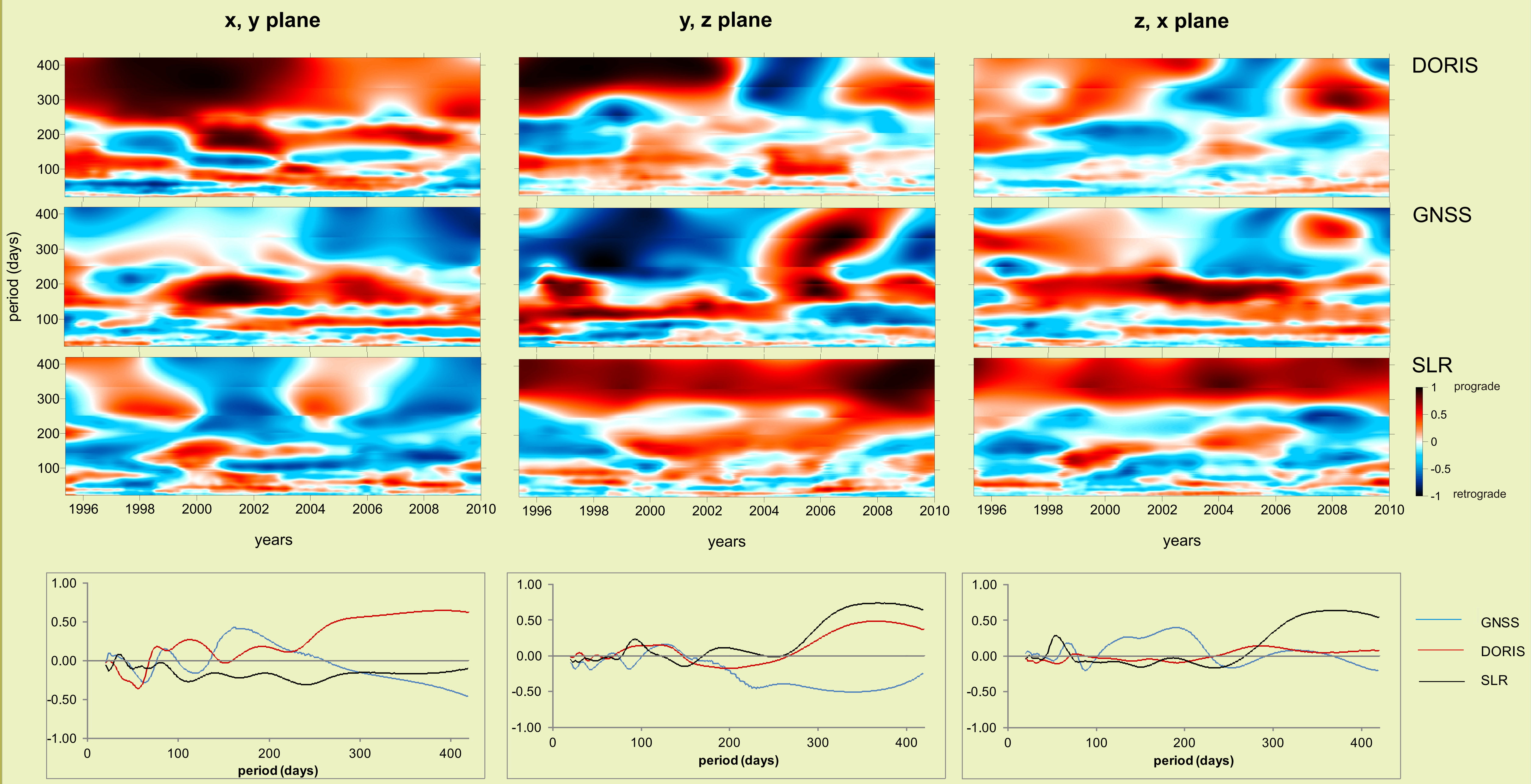


Fig. 3. The spectro-temporal polarization functions (1.0, m 500) and the mean polarization functions (1.0) of the DORIS, GNSS and SLR center

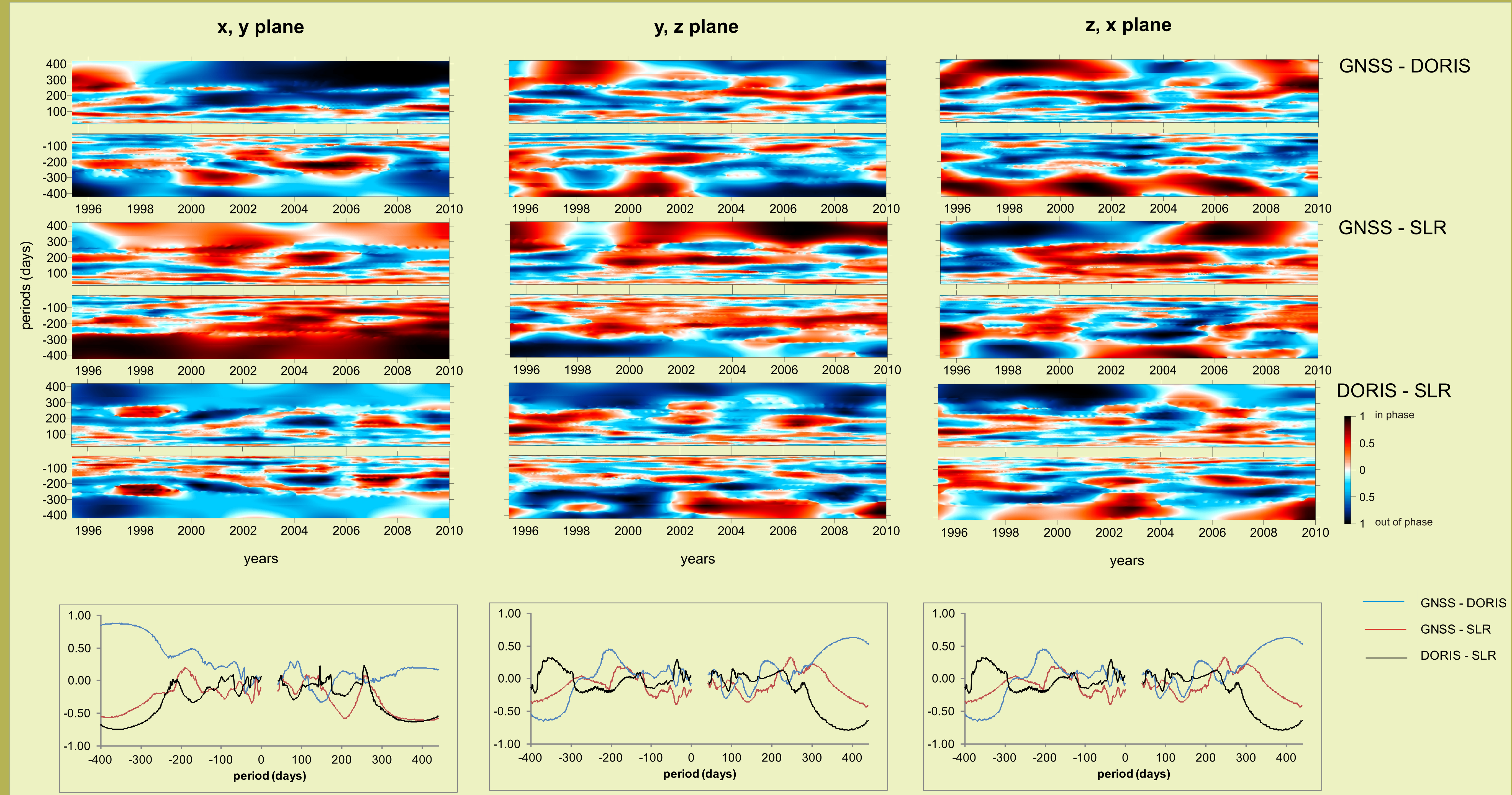


Fig. 4. The spectro-temporal semblance functions (1.0, m 500) and the mean semblance functions (1.0) of the DORIS, GNSS and SLR center c

CONCLUSION

The spectra of DORIS geocenter time series show domination of noise in these data in the short period band including periods less than half a year.

In the equatorial xy plane the annual oscillation in DORIS geocenter time series is mostly prograde while it is mostly retrograde in SLR and GNSS geocenter time series. The semblance functions between GNSS and SLR geocenter time series show good phase agreement in the retrograde annual oscillation in the equatorial plane and disagreement of prograde and retrograde annual oscillations of these two techniques with DORIS.

The annual oscillation in SLR geocenter time series is mostly prograde i yz and zx planes.

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ACKNOWLEDGEMENTS

We would like to thank dr K. Sośnica for providing the SLR centre of mass data as well as dr Robert Weber, dr Jim Ray and dr Paul Rebischung for providing weekly IGS SINEX combinations of GNSS centre of mass time series. The financial supports of the Agriculture University in Krakow.